

# **Influence of Time Lag and Noncolocation on Integrated Structural/Control System Designs<sup>†</sup>**

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## Introduction

Recent research efforts have led to the development of simultaneous structural/control system design procedures [1–3]. Absent in any of this work is the time delay present in the control system sensors and actuators and the computational time delay for synthesizing actuator commands from sensor measurements. Madden [4] has shown that the time delay present in the control system can have profound effects on the resulting system performance and stability regardless of its source. In addition, many of the simultaneous structural/control system design procedures have used colocated sensors and actuators for implementation of the control system. In actual practice, colocation is not always possible (e.g., actuator output forces are based on optical quality measurements such as line-of-sight). Spector and Flashner [5] and Bong Wei [6] have raised the issue of stability degradation when using noncolocated sensor and actuators.

This work extends the integrated structural/control system design procedure reported in Reference [3] to include the effects of time lag and noncolocation of sensors and actuators on the resulting optimum designs.

### Optimum Design Problem Statement

In Reference [3] the integrated controls/structure optimum design problem was posed as either the mass minimization problem given in equations (1) through (4) or the control effort minimization problem given in equations (5) through (8). The set of behavior constraints,  $g_m(d, t)$ , consists of time parametric upper bounds on the peak transient dynamic displacements and accelerations at selected degrees-of-freedom as well as upper or lower bounds on selected natural frequencies. The vector of design variables,  $d$ , consists of finite element box beam cross-sectional dimensions, spherical nonstructural mass element radii, and nonlinear on/off control system velocity thresholds and actuator output force magnitudes.

The solution to either the mass minimization problem or the control effort minimization problem is found by solving a sequence of explicit approximate problems. Each approximate problem is constructed using first order hybrid approximations for all of the critical (or near critical) behavior constraints as well as for the control effort objective function in (5) or upper bound constraint in (3). Time parametric peak transient dynamic displacement and/or acceleration sensitivities for use in the hybrid approximations are calculated in an efficient manner using the Wilkie-Perkins essential parameter sensitivity method [7, 8] which significantly reduces the amount of time stepping needed to obtain these sensitivities.

$$\min W(d) \quad (1)$$

$$\text{subject to } g_m(d, t) \geq 0 \quad (2)$$

$$E(d, t_f) \leq E^u \quad (3)$$

$$d_j^l \leq d_j \leq d_j^u \quad (4)$$

or

$$\min E(d, t_f) \quad (5)$$

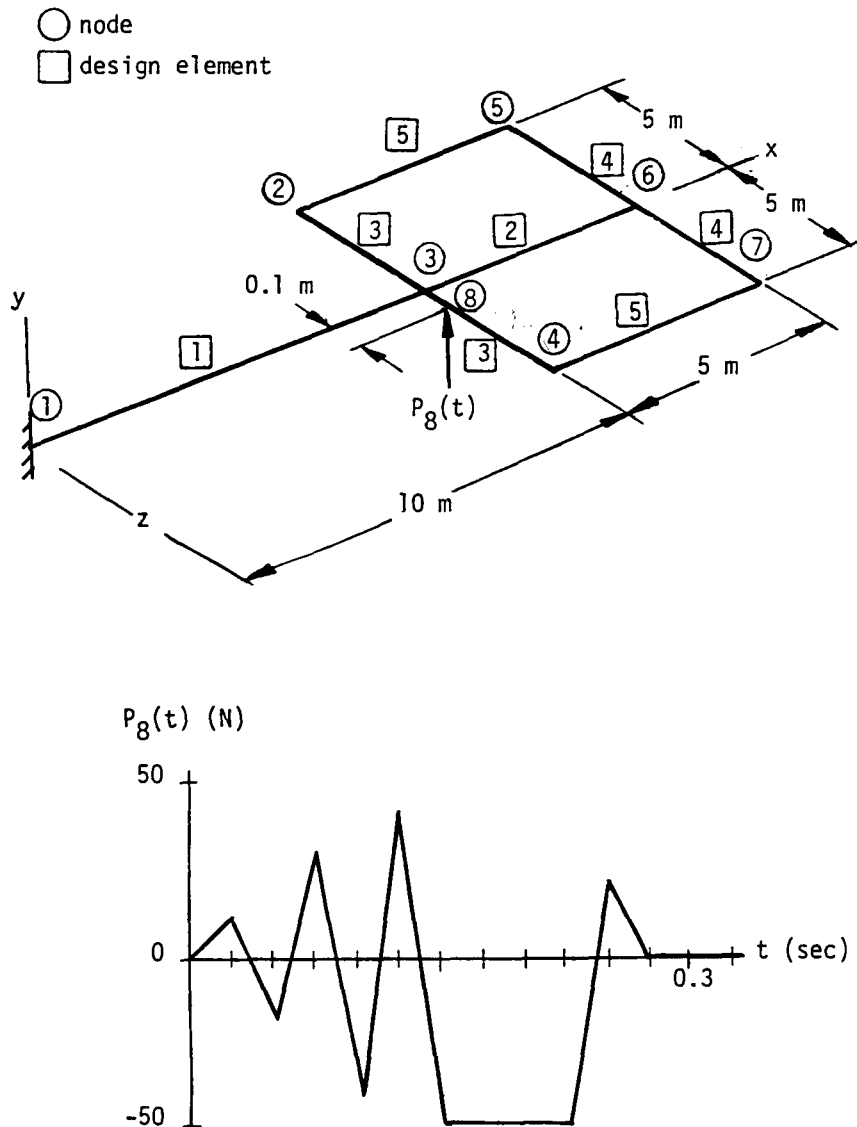
$$\text{subject to } g_m(d, t) \geq 0 \quad (6)$$

$$W(d) \leq W^u \quad (7)$$

$$d_j^l \leq d_j \leq d_j^u \quad (8)$$

## Numerical Example

The 21 degree-of-freedom aluminum grillage structure shown is used to examine the effects of control system delay and noncolocation of sensors and actuators on the resulting optimum designs. Nine analysis box beam finite elements are linked to yield five design elements. The external load shown was applied at node 8 of the structure so as to excite both cantilever bending modes and torsional modes. A colocated sensor/actuator pair is located at node 6 of the structure to try and reduce dynamic response. Upper bounds of  $9.0 \times 10^{-4} \text{ m}$  were placed on the peak dynamic displacement response at nodes 5, 6, and 7. All dynamic response calculations were carried out for 1 second using 10 retained modes (frequency content up to 100 Hz), 2% modal damping, and a time step of 0.005 seconds. Both the minimum weight problem (with an upper bound placed on control effort) and the minimum control effort problem (with a weight cap) were used to demonstrate the effects of control system time lag on the resulting optimum designs. Results for this problem were reported in Reference [3] using colocated sensors and actuators and no time lag.



## Numerical Results - With Time Lag

The actuator output forces were represented as exponential growth functions with the value of the time lag defined as the time it takes to generate 90% of the maximum actuator output force from the time the actuator is commanded to generate force. For each value of time lag, a complete optimization was done from the same starting point. The results of the optimizations for each time lag are shown in the accompanying tables and figures.

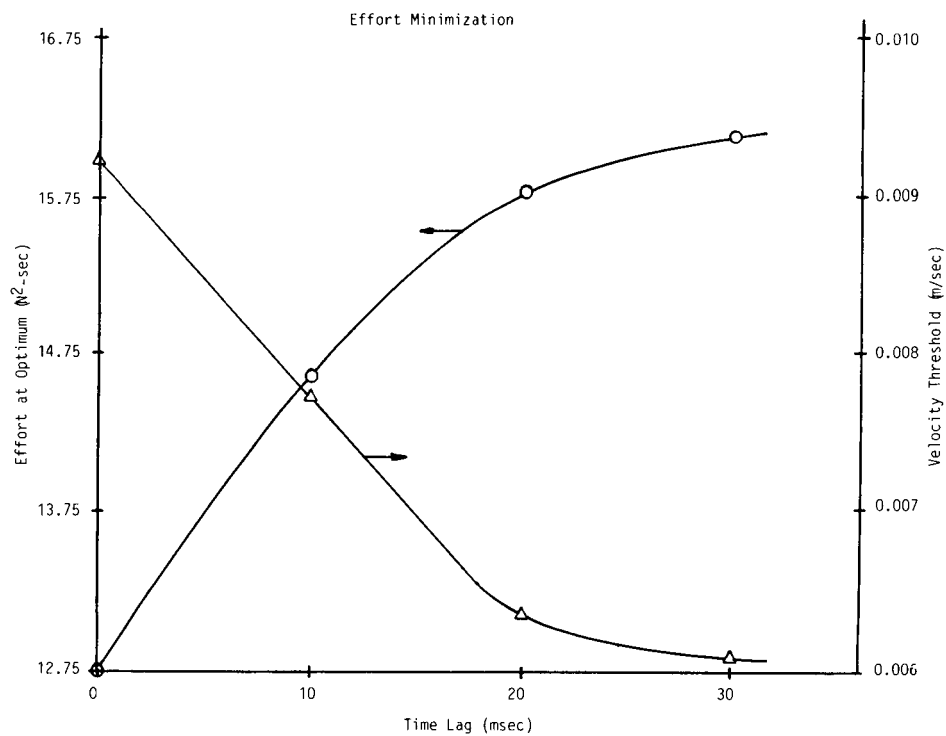
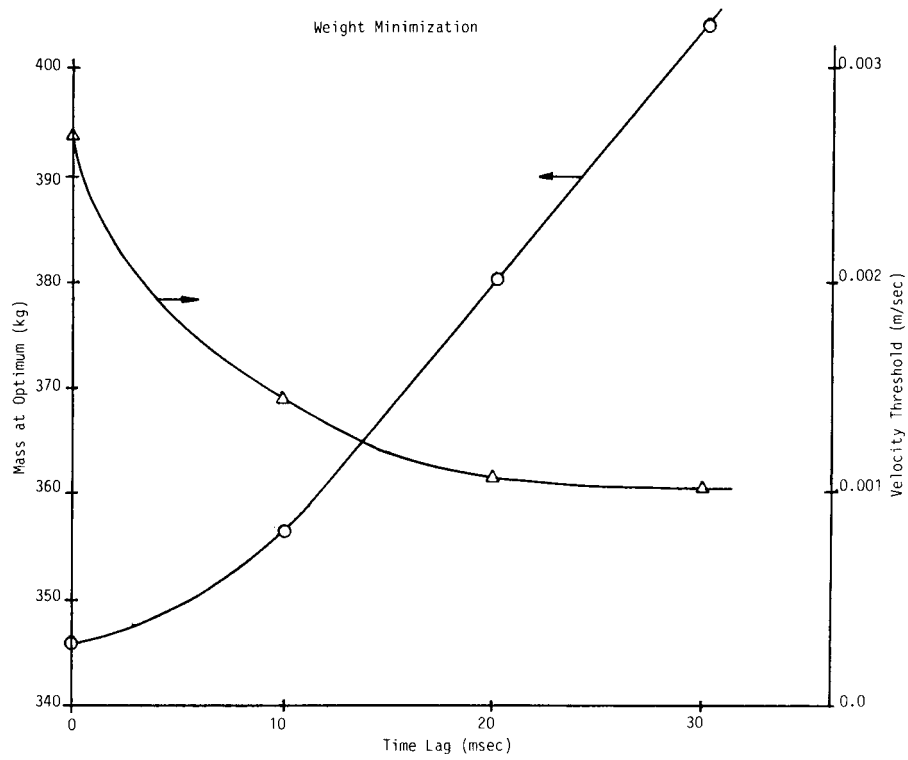
The figures show that the value of the objective function increases with increasing time lag when displacement constraints drive the optimum design. Also plotted in the figures are the corresponding values of the velocity threshold design variable,  $\epsilon_i$ . One can see that as the time lag increases, the value of the velocity threshold at the optimum design decreases. This indicates that the optimizer is compensating for the time lag by commanding the actuators to generate force sooner. At time lag values greater than 15 milliseconds the minimum weight increases nearly linearly since the velocity threshold design variable is against its lower bound side constraint.

Time histories for the critical constraints and corresponding actuator forces are shown on the following pages.

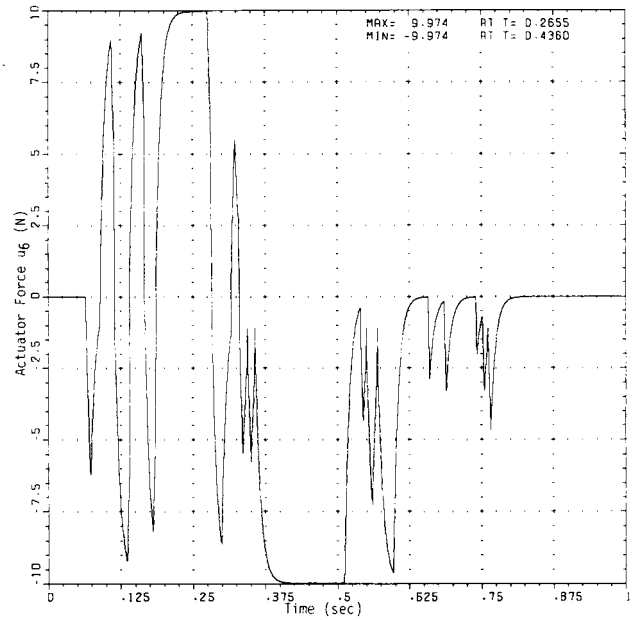
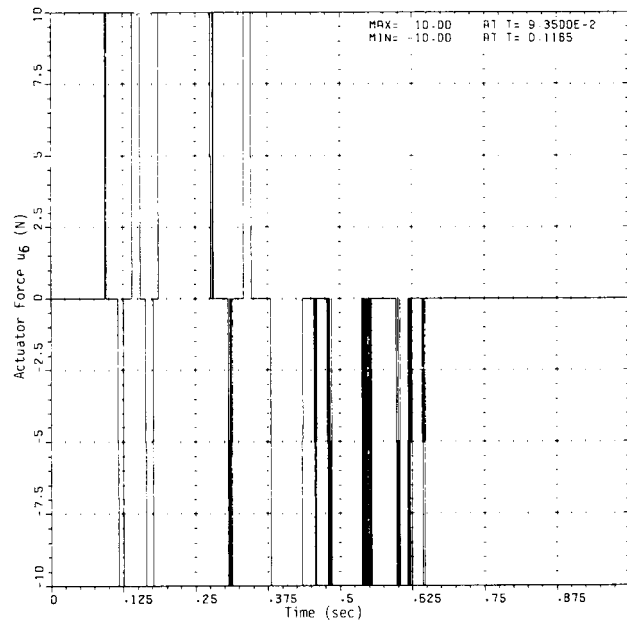
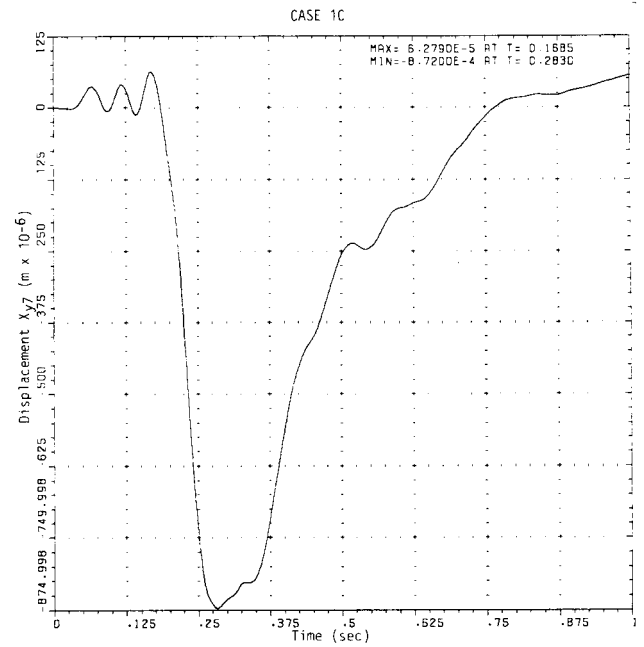
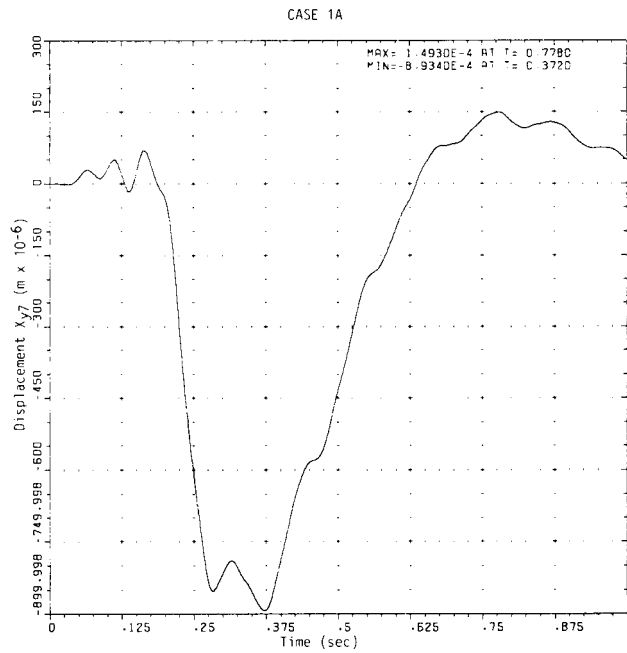
Table 1: Effect of Time Lag on the Optimum Designs

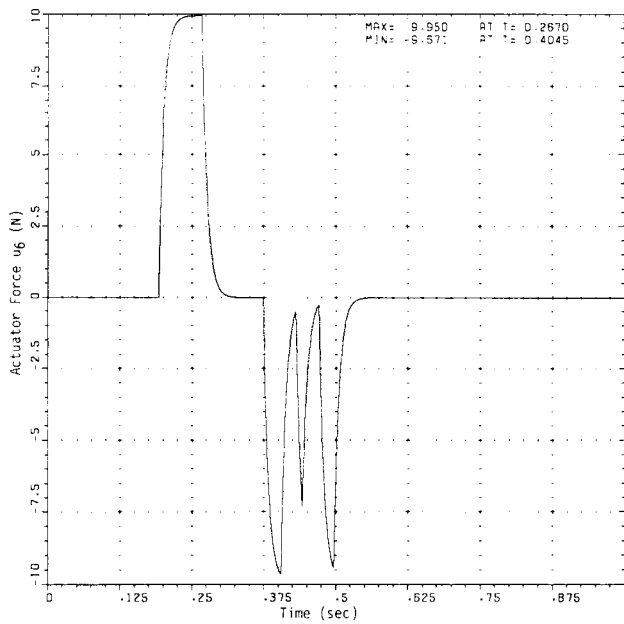
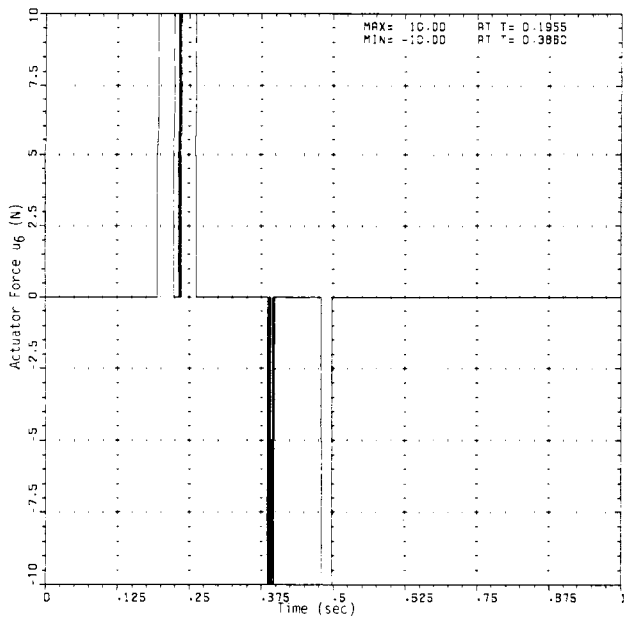
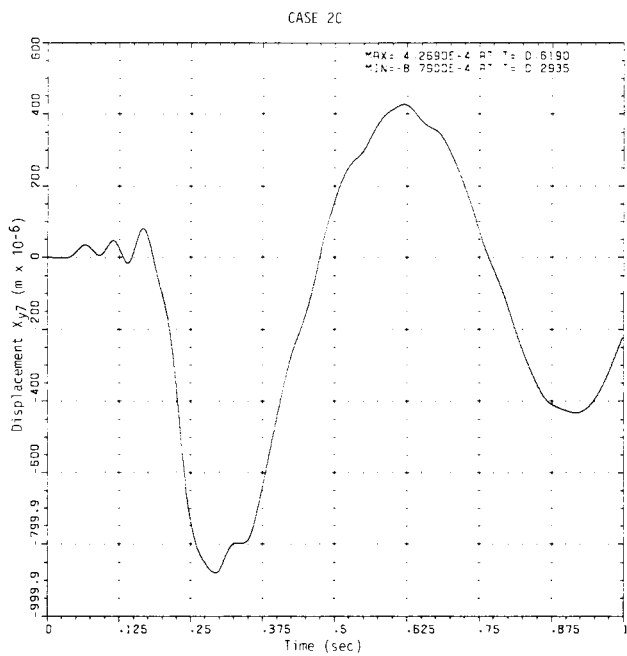
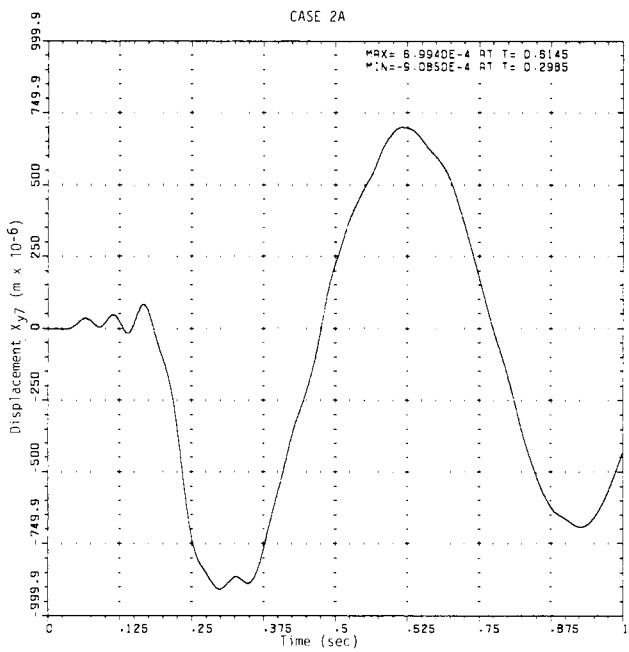
Case	Objective Function	Time Lag (sec)	Objective Function Value	Critical Behavior Constraints
1A	mass	0.0001	346	$X_{y7}$
1B	mass	0.0010	357	$X_{y7}$
1C	mass	0.0020	381	$X_{y7}$
1D	mass	0.0030	404	$X_{y7}$
2A	effort	0.0001	12.75	$X_{y6}, X_{y7}, W$
2B	effort	0.0010	14.65	$X_{y6}, X_{y7}, W$
2C	effort	0.0020	15.85	$X_{y6}, X_{y7}, W$
2D	effort	0.0030	16.15	$X_{y6}, X_{y7}, W$

## Numerical Results - With Time Lag



# Numerical Results - With Time Lag





## Numerical Results - Noncolocated Sensors and Actuators

The minimum weight design problem was run again using noncolocated sensors and actuators without time lag. In this case, simple estimators (based on kinematics) had to be constructed to generate velocities at actuators locations from the measured velocities at sensor locations. Case 3A in the table below gives the result obtained in Reference [3] for colocated sensors and actuators. The two configurations of noncolocated sensor/actuator locations, their respective estimators, and the resulting optimum designs are given in the table as cases 3B and 3C.

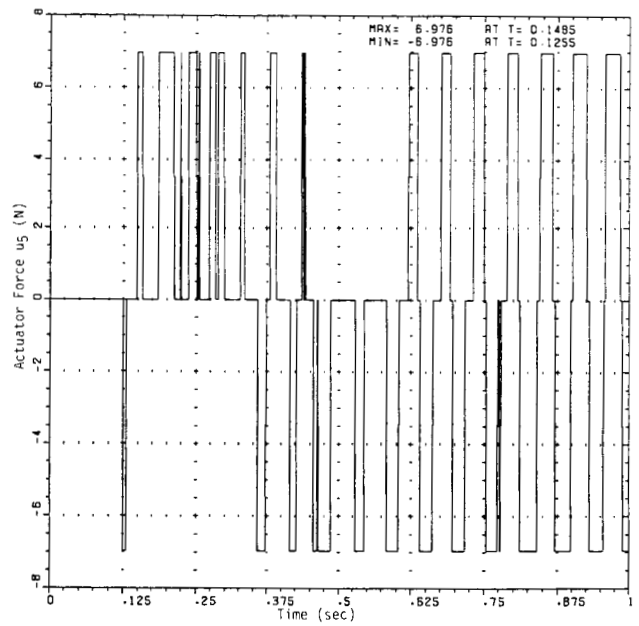
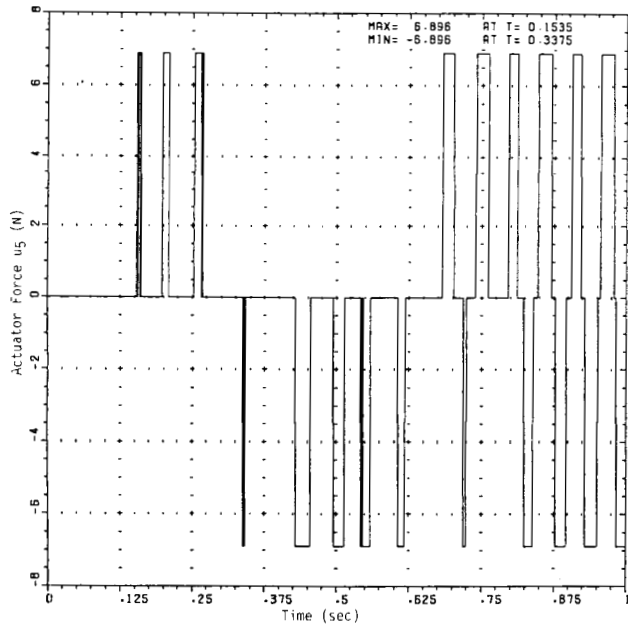
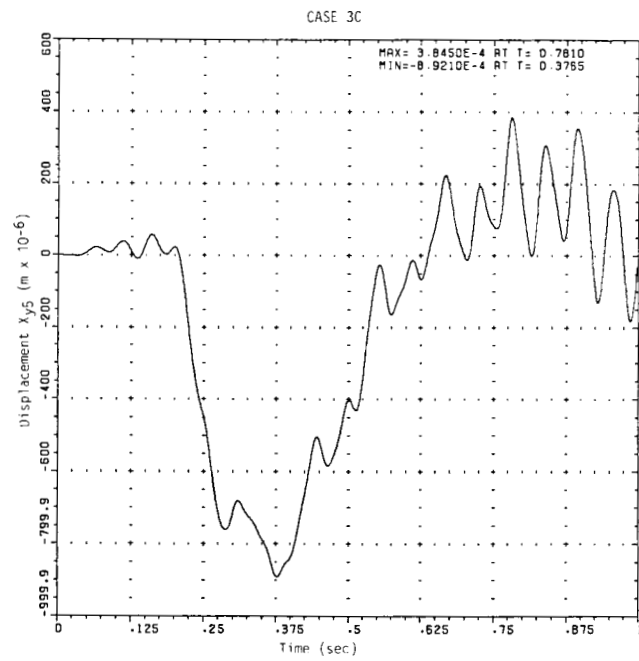
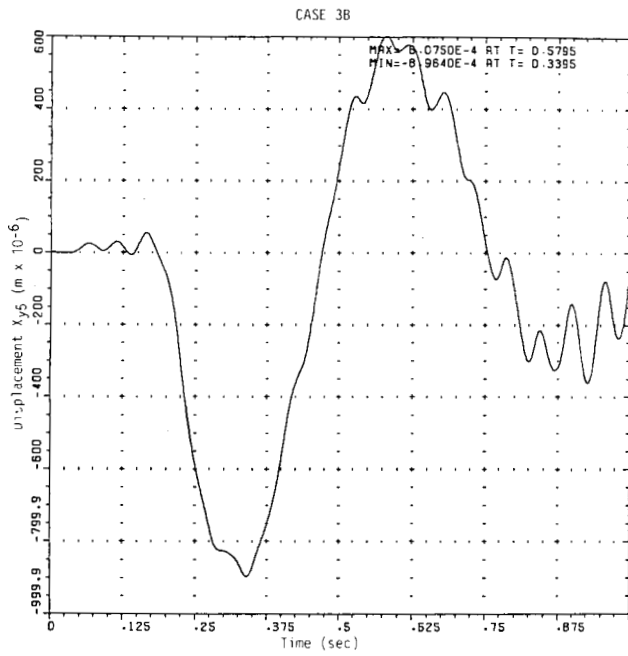
Case 3B and 3C time histories for the critical constraint at the optimum designs,  $X_{y5}$ , along with the actuator force outputs are shown in the accompanying figures. In both instances, peak dynamic displacements are reduced in the time period considered when compared with the uncontrolled case. However, towards the end of the time history, both cases show high frequency oscillations superimposed on the response. In fact, both responses are unstable if carried out for a longer period of time. This instability is caused entirely by the fifth mode (17.8 Hz in case 3B, 18.2 Hz in case 3C) where the sign of the translational component of the eigenvector at the sensor location is opposite to the sign at the actuator locations. As pointed out by Bong Wie [6], systems with noncolocated sensors and actuators tend to be closer to the stability bounds for just that reason. The actuator force time histories show that the control system is pumping energy into the system at exactly the frequency of the fifth mode, thus driving it unstable.

Table 2: Effect of Noncolocation on the Optimum Designs

Case	Nodal Sensor Locations	Nodal Actuator Locations	Estimator Equations	Objective Function Values
3A	5,7	5,7		259
3B	6	5,7	$X_{y5} = X_{y6} + 5X_{\theta_x6}$ $X_{y7} = X_{y6} - 5X_{\theta_x6}$	406
3C	3	5,7	$X_{y5} = X_{y3} + 5X_{\theta_x3} + 5X_{\theta_z3}$ $X_{y7} = X_{y3} - 5X_{\theta_x3} + 5X_{\theta_z3}$	423



## Numerical Results - Noncolocated Sensors and Actuators



## Concluding Remarks

The effects of including time lag and allowing noncollocated sensors and actuators on optimum designs has been explored. Results of this study show that neglect of time lag in the control system can lead to unconservative designs (i.e., lower objective function designs than can be physically realized). However, the time lag results indicate that it is feasible to incorporate this refinement within a design optimization procedure which includes direct constraints on peak transient dynamic displacements at specified degrees of freedom. Furthermore, it is found that the optimization procedure compensates for the presence of time lag in the system by lowering the velocity thresholds, thus turning on the control system sooner. On the other hand, the use of noncollocated sensors and actuators can result in convergence to a dynamically unstable system when trying to control modes where the sensors and actuators are out of phase with respect to one another. The results for the noncollocated sensors and actuators indicate that it will be necessary to add constraints on appropriate dynamic stability measures in order to prevent unstable behavior.

## References

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